Transposition of formulae: the challenge of not 'moving'

One sunny afternoon two new mothers, coincidentally Maths Educators, met in a cafe, their babies sleeping peacefully in strollers[1]. Over tea, the conversation evolved from baby food and sleeping patterns, to life in general and onto work matters. You see, prior to their maternity leave, both Julie and Catherine happened to be involved with the Academic Learning Centre at Cork Institute of Technology, helping students with all maths matters. The two were wondering why some topics were particularly hard for the students and how rearranging equations (also known as transposition of formulae) was one of the milestones many just could not grasp.

To a maths educator, the concept of transposition seems well defined and methodical: do the same to both sides of the equation and apply inverse operations to remove various entities until the subject of the equation is isolated. ~~Yet the untold horrors observed in students' work is enough to make your eyes bleed.~~ Yet the calamity of errors observed in students' work is alarming. Some of the errors observed do not make any sense and cannot be explained by algebraic misconceptions alone. Why are students 'mutilating' equations in such strange ways?! What kind of thought process do they follow and how do they justify it?

While analysing student errors, as well as talking to the students, it became apparent that some are not applying any concepts and principles of rearranging equations. Instead, they are 'moving' entities across equations in a random fashion (see some examples in the figure). When asked about this the students often have a rather bewildered tone to their reply: "isn't this what you are supposed to do?" How do they come to think that this is the way?

$$60 + r = 0 \Rightarrow r = 60$$

$$\frac{60}{r} + 10 = 1 \Rightarrow 70 = r$$

While talking to colleagues, we realised that there is significant variation in the language we use around rearranging equations. Indeed some of us habitually use the words 'moving', 'shifting', 'bringing across'. Could it be that this terminology, when used in class, and heard over and over again by the students, seeds the wrong idea of the principles of transposition? It seems that it does, and especially so with the students who have not yet grasped the concept of transposition fully.

Following this realisation, we and other educators in the department agreed to 'watch our language'. The table below shows how the same transposition problem can be worked through with and without the use of unhelpful language. We are now actively trying to avoid words such as 'move', 'shift', 'bring over' and similar but instead use '**get rid of**', '**remove'**, '**have … on the other side of equation**', **'add … to both sides of the equation'**, **'divide both sides of the equation by …**'. The bottom line is: we do not 'move' anything. We do the same thing to both sides of an equation in order to 'get rid of' things which are in the way. In addition to

[1] Okay okay! Perhaps the part about the babies sleeping peacefully in strollers is not entirely true!

using appropriate language ourselves, we also step in and correct a student who is using misleading language in class to prevent the spread of unhelpful language amongst their peers.

Scenario 1: unhelpful language	Whiteboard	Scenario 2 : language that is not misleading
"I **move** 5 to the left side and it changes the sign".	$b = 2a + 5 \qquad a = ?$	"I want to **have 5 on the left side** of equation. What should I do? **Subtract** 5 from **both sides**."
"We **bring the 2 across** the equals sign and divide by it".	$b - 5 = 2a$	"Now I want to **get rid of** 2. How do I **remove multiplication** by 2? **Divide both sides** by 2."
	$\frac{b-5}{2} = a$	

We recognise that the use of appropriate language will not solve all problems surrounding the topic of transposition but hope that it will prevent some misconceptions and will open up space for the true concepts to settle in. We also realise that our efforts alone cannot effect a significant change. When students enter third level college, the idea of 'moving' can already be firmly formed in their minds and such habits can be very hard, if not impossible, to break. The process of change needs to start much earlier than college, in school.

Dear fellow Maths Educators, we are optimistic that together we can make a difference, and we call upon you to accept this challenge and join us in adopting the 'good language of transposition'. It is only for the benefit of all students, their professional careers and our future as a numerate society!

If you have any comments or would like to discuss this topic further please get in touch via email catherine.palmer@cit.ie. More information on our work on transposition of formulae is here.

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P.S.

Joe: Alright biy? How's it going?

Liam: It's this maths homework. It's wrecking my head. How do you move that 5 to the other side?

Joe: You don't move it, biy! You multiply both sides by 5 and then it's gone from here.

Liam: Is that all you do?!!

Joe: Yea biy! Keep calm and do the same thing to both sides!